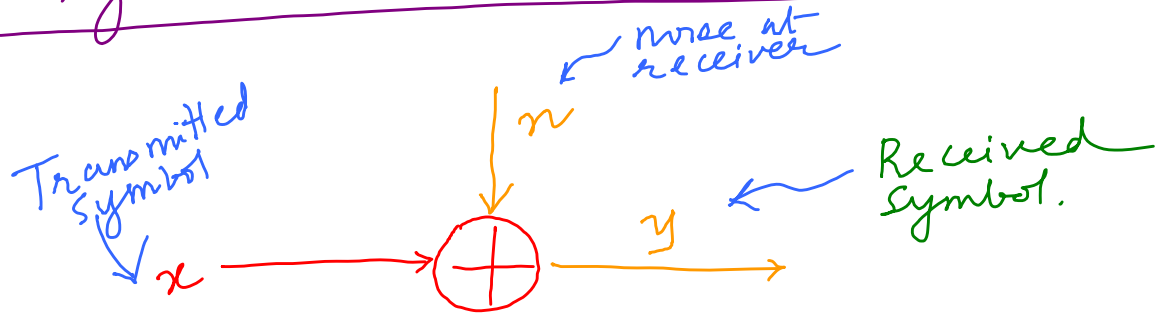


Performance of Wireline Communication System :



Model for a Basic wireline system

$$y = x + n$$

AWGN
Additive white Gaussian noise

white Gaussian

Additive noise

Noise PDF

$$f_N(n) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-n^2/2\sigma^2}$$

$$N(0, \sigma^2)$$

mean = 0

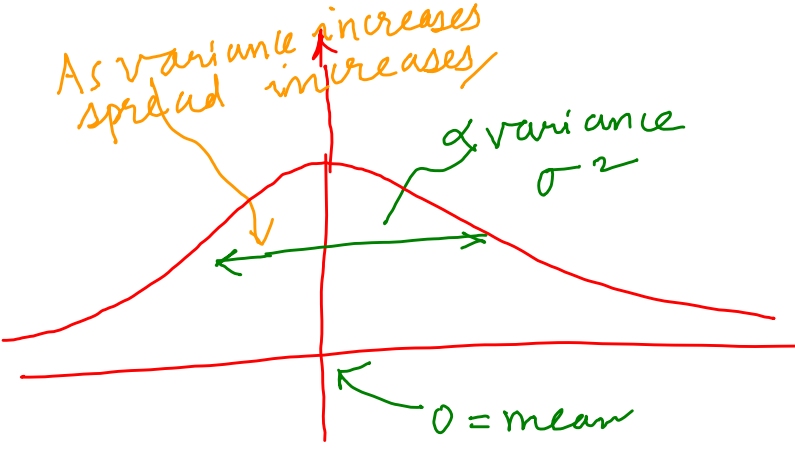
variance = σ^2

Noise power = σ^2

As variance increases spread increases

variance σ^2

0 = mean



Consider $x = 1 = -\sqrt{P}$

$$\begin{aligned} y &= x + n \\ &= -\sqrt{P} + n \end{aligned}$$

Bit error occurs if $y \geq 0$

$$\begin{aligned} &\Rightarrow -\sqrt{P} + n \geq 0 \\ &\Rightarrow n \geq \sqrt{P} \end{aligned}$$

Probability of Error

$$= P(n \geq \sqrt{P})$$

$$= \int_{\sqrt{P}}^{\infty} f_N(n) dn = \int_{\frac{\sqrt{P}}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{t^2}{2}} \sigma dt$$
$$= \int_{\sqrt{P}}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{n^2}{2\sigma^2}} dn$$
$$\frac{n}{\sigma} = t \quad dn = \sigma dt$$

$$P_e = P(n > \sqrt{P})$$

$$= \int_{\frac{\sqrt{P}}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$$


 Gaussian RV
with mean = 0
variance = 1

$$= P\left(\mathcal{N}(0, 1) \geq \sqrt{\frac{P}{\sigma^2}}\right)$$

$$P_e = Q\left(\sqrt{\frac{P}{\sigma^2}}\right)$$

$$Q(v) = \int_v^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$$

Probability of Bit Error in
AWGN channel

$$= Q\left(\sqrt{\frac{P}{\sigma^2}}\right) = Q\left(\sqrt{\text{SNR}}\right)$$

For BPSK modulated
Transmission of average
power = P

$$\frac{P}{\sigma^2} = \text{SNR}$$

Signal to Noise
Power Ratio
of System

Example: At $\text{SNR} = 10 \text{ dB}$, what is the BER for our AWGN communication system, with BPSK modulation?

$$10 \log_{10} \text{SNR} = 10 \text{ dB}$$

$$\log_{10} \text{SNR} = 1$$

$$\text{SNR} = 10^1 = 10$$

Prob of Bit error
For BPSK over
AWGN channel
at $\text{SNR} = 10 \text{ dB}$

$$P_e = Q(\sqrt{\text{SNR}}) = Q(\sqrt{10})$$

$$P_e = 7.82 \times 10^{-4}$$

$$Q(x) \leq \frac{1}{2} e^{-\frac{1}{2}x^2}$$

Convenient Approximation

$$Q(x) \approx \frac{1}{2} e^{-\frac{1}{2}x^2}$$

$$Q(\sqrt{\text{SNR}}) \approx \frac{1}{2} e^{-\frac{1}{2}\text{SNR}}$$

Approximation
of BER for
BPSK over
AWGN

$$\text{BER} = \frac{1}{2} e^{-\frac{1}{2} \cdot 10}$$

$$= \frac{1}{2} e^{-5} = \underline{3.91 \times 10^{-4}}$$

Approximate BER
at SNR = 10 dB



Example: What is SNR_{dB} required
for $\text{BER} = 10^{-6}$? BPSK over AWGN

$$\frac{1}{2} e^{-\frac{1}{2} \text{SNR}} = 10^{-6}$$

$$\Rightarrow e^{-\frac{1}{2} \text{SNR}} = 2 \times 10^{-6}$$

$$\Rightarrow \text{SNR} = -2 \times \ln 2 \times 10^{-6} \\ = 26.24$$

$$\text{SNR}_{\text{dB}} = 10 \log_{10}(26.24)$$

$$\boxed{\text{SNR}_{\text{dB}} = 14.19 \text{ dB}}$$